The definite integral and Applications

1. Compute the definite integrals below.

a. $\displaystyle\int_0^2 2x^3 + x^2 - 5x + 2\,dx =$

d. $\displaystyle\int_0^{20} 500e^{-0.04t}\,dt =$

b. $\displaystyle\int_1^8 2\sqrt[3]{t} + \frac{3}{\sqrt[3]{t^2}}\,dt =$

e. $\displaystyle\int_0^4 3t\sqrt{t^2 + 9}\,dt$

c. $\displaystyle\int_0^4 \frac{5}{4x + 1}\,dx =$

2. Find the area of the region bounded by the graphs $y = 2\sqrt{x}$ and $y = 1 - 2x$, and the lines $x = 1$ and $x = 4$.

3. What are the *Producers' surplus* and *Consumers' surplus* for the market with supply function
$$p = 0.05q^2 + 3q + 5$$
and demand function
$$p = 100 - 0.75q.$$

4. Find the average value of the function $f(x) = \dfrac{x^4 - 1}{x^2}$ on the interval $[1, 3]$.

5. Find the Gini coefficient of inequality for the nation with income distribution curve
$$y = 0.5x^3 + 0.3x^2 + 0.2x,$$
where $y \cdot 100\%$ is the percentage of national income earned by the poorest $x \cdot 100\%$ of the population.

6. The marginal propensity to save of a small nation is given by
$$\frac{dS}{dY} = \frac{Y + 5}{9Y + 10},$$
where savings S and national income Y are both measured in billions of dollars. Express the total change in national savings when income increases from \$10 billion to \$15 billion as a definite integral, and find its value. What is the total change in national consumption?