1. The demand function for a firm's product is given by $q = 6\ln(5y - 0.5p)$, where

- q is the monthly demand for the firm's product, measured in 1000's of units;
- y is the average monthly disposable income in the market for the firm's product measured in 1000s of dollars;
- p is the price of the firm's product, also measured in dollars.

(a) (5 pts) Find q, $\partial q/\partial y$ and $\partial q/\partial p$ when the monthly income is \$3000 and $p = 10$. Round your (final) answers to two decimal places.

Since income is measured in \$1000*s, an average monthly income of* \$3000*, means that* $y = 3\ldots$

$$q\Big|_{\substack{y=3\\p=10}} = 6\ln 10 \approx 13.816, \qquad \frac{\partial q}{\partial y}\Big|_{\substack{y=3\\p=10}} = \frac{30}{5y-0.5p}\Big|_{\substack{y=3\\p=10}} = \frac{30}{10} = 3$$

and

$$\frac{\partial q}{\partial p}\Big|_{\substack{y=3\\p=10}} = -\frac{3}{5y-0.5p}\Big|_{\substack{y=3\\p=10}} = -\frac{3}{10} = -0.3.$$

(b) (3 pts) Use *linear approximation* and your answers to (a) to ***estimate*** the change in monthly demand for the firm's product if the average income in the market increases to \$3200 and the price of the firm's product increases to \$10.5.

Once again, since income is measured in \$1000*s, a change of* \$200 *means that* $\Delta y = \frac{200}{1000} = 0.2$*, and linear approximation says that...*

$$\Delta q \approx \left(\frac{\partial q}{\partial y}\Big|_{\substack{y=3\\p=10}}\right)\cdot \Delta y + \left(\frac{\partial q}{\partial p}\Big|_{\substack{y=3\\p=10}}\right)\cdot \Delta p = 3\cdot(0.2) + (-0.3)\cdot(0.5) = 0.45$$

i.e., demand will increase by about 0.45 *(which is* 450 *units, since demand is measured in* 1000*s of units).*

(c) (2 pts) Find the price-elasticity of demand for the firm's product when the price is \$10 and average income is \$3000.

The price-elasticity of demand is found using the formula $\eta = \frac{\partial q}{\partial p}\cdot\frac{p}{q}$*, so that the given point...*

$$\eta\Big|_{\substack{y=3\\p=10}} = \frac{\partial q}{\partial p}\cdot\frac{p}{q}\Big|_{\substack{y=3\\p=10}} = (-0.3)\cdot\frac{10}{6\ln 10} \approx -0.217$$

2. (10 pts) Find the critical point(s) and critical value(s) of the function

$$f(x,y) = 2x^3 - 6xy + y^2 + 1,$$

and use the second derivative test to classify the critical value(s) as relative minimum value(s), relative maximum value(s) or neither.

First we solve the first-order equations:

$$\begin{aligned} f_x = 0 \quad &\Longrightarrow \quad 6x^2 - 6y = 0 \\ f_y = 0 \quad &\Longrightarrow \quad -6x + 2y = 0 \end{aligned}$$

The equation $f_x = 0$ shows that at the critical points $6y = 6x^2$ so $y = x^2$. Substituting this into the equation $f_y = 0$ gives the equation

$$-6x + 2x^2 = 0 \implies 2x(x-3) = 0.$$

This means that the two critical x values are $x_1 = 0$ and $x_2 = 3$. The corresponding critical y-values are $y_1 = x_1^2 = 0$ and $y_2 = x_2^2 = 9$, so the critical points are $(0,0)$ and $(3,9)$. The critical values are $f(0,0) = 1$ and $f(3,9) = -26$.

Next, we find the second derivatives of $f(x,y)$ and the discriminant to apply the second derivative test:

$$f_{xx} = 12x,\ f_{xy} = -6, \quad \textit{and} \quad f_{yy} = 2 \implies D = f_{xx}f_{yy} - f_{xy}^2 = 24x - 36.$$

(*) At the critical point $(0,0)$ we have $D(0,0) = -36 < 0$ so the critical value $f(0,0) = 1$ is ***neither a minimum nor a maximum****.*

(*) At the critical point $(3,9)$ we have $D(3,9) = 36 > 0$ and $f_{xx}(3,9) = 36 > 0$, so the critical value $f(3,9) = -26$ is a ***relative minimum*** *value.*