1. The demand function for a firm's product is given by $q = 8\ln(6y - 0.75p)$, where

- q is the monthly demand for the firm's product, measured in 1000's of units;
- y is the average monthly disposable income in the market for the firm's product measured in 1000s of dollars;
- p is the price of the firm's product, also measured in dollars.

(a) (5 pts) Find q, $\partial q/\partial y$ and $\partial q/\partial p$ when the monthly income is \$3500 and $p = 12$. Round your (final) answers to two decimal places.

Since income is measured in \$1000s, an average monthly income of \$3500, means that $y = 3.5$...

$$q\bigg|_{\substack{y=3.5\\p=12}} = 8\ln 12 \approx 19.878, \qquad \frac{\partial q}{\partial y}\bigg|_{\substack{y=3.5\\p=12}} = \frac{48}{6y - 0.75p}\bigg|_{\substack{y=3.5\\p=12}} = \frac{48}{12} = 4$$

and

$$\frac{\partial q}{\partial p}\bigg|_{\substack{y=3.5\\p=12}} = -\frac{6}{6y - 0.75p}\bigg|_{\substack{y=3.5\\p=12}} = -\frac{6}{12} = -0.5.$$

(b) (3 pts) Use *linear approximation* and your answers to (a) to ***estimate*** the change in monthly demand for the firm's product if the average income in the market increases to \$3800 and the price of the firm's product increases to \$12.40.

Once again, since income is measured in \$1000s, a change of \$300 means that $\Delta y = \frac{300}{1000} = 0.3$, and linear approximation says that...

$$\Delta q \approx \left(\frac{\partial q}{\partial y}\bigg|_{\substack{y=3.5\\p=12}}\right) \cdot \Delta y + \left(\frac{\partial q}{\partial p}\bigg|_{\substack{y=3.5\\p=12}}\right) \cdot \Delta p = 4 \cdot (0.3) + (-0.5) \cdot (0.4) = 1$$

i.e., demand will increase by about 1 (which is 1000 units, since demand is measured in 1000s of units).

(c) (2 pts) Find the price-elasticity of demand for the firm's product when the price is \$12 and average income is \$3500.

The price-elasticity of demand is found using the formula $\eta = \frac{\partial q}{\partial p} \cdot \frac{p}{q}$, so that the given point...

$$\eta\bigg|_{\substack{y=3.5\\p=12}} = \frac{\partial q}{\partial p} \cdot \frac{p}{q}\bigg|_{\substack{y=3.5\\p=12}} = (-0.5) \cdot \frac{12}{8\ln 12} \approx -0.302$$

2. (10 pts) Find the critical point(s) and critical value(s) of the function

$$g(u, v) = 3u^2 + 6uv - 2v^3 + 7,$$

and use the second derivative test to classify the critical value(s) as relative minimum value(s), relative maximum value(s) or neither.

First we solve the first-order equations:

$$\begin{aligned} g_u = 0 \quad &\Longrightarrow \quad 6u + 6v &= 0 \\ g_v = 0 \quad &\Longrightarrow \quad 6u - 6v^2 &= 0 \end{aligned}$$

The equation $g_v = 0$ shows that at the critical points $6u = 6v^2$ so $u = v^2$. Substituting this into the equation $g_u = 0$ gives the equation

$$6v^2 + 6v = 0 \implies 6v(v + 1) = 0.$$

This means that the two critical v values are $v_1 = 0$ and $v_2 = -1$. The corresponding critical u-values are $u_1 = v_1^2 = 0$ and $u_2 = v_2^2 = 1$, so the critical points are $(0,0)$ and $(1,-1)$. The critical values are $g(0,0) = 7$ and $g(1,-1) = 6$.

Next, we find the second derivatives of $g(u,v)$ and the discriminant to apply the second derivative test:

$$g_{uu} = 6,\ g_{uv} = 6, \quad \textit{and} \quad f_{vv} = -12v \implies D = g_{uu}g_{vv} - g_{uv}^2 = -72v - 36.$$

*(*) At the critical point $(0,0)$ we have $D(0,0) = -36 < 0$ so the critical value $g(0,0) = 7$ is **neither a minimum nor a maximum**.*

*(*) At the critical point $(1,-1)$ we have $D(1,-1) = 36 > 0$ and $g_{uu}(1,-1) = 6 > 0$, so the critical value $g(1,-1) = 6$ is a **relative minimum** value.*