1. (4 pts) $\displaystyle\int_0^4 \frac{5\,dx}{\sqrt{x^2+9}} = 5\cdot\ln\left|x+\sqrt{x^2+9}\right|\Big|_0^4 = 5(\ln 9 - \ln 3) = 5\ln(9/3) = 5\ln 3 \quad (\approx 5.493)$

Using the formula $\displaystyle\int \frac{du}{\sqrt{u^2+a^2}} = \ln\left|u+\sqrt{u^2+a^2}\right| + C$, with $a^2 = 9$.

2. (6 pts) Find the present value of a continuous income stream that pays at the annual rate $f(t) = 1000t$ for $T = 10$ years, assuming that the interest rate is $r = 3\%$.

$$\begin{aligned}\text{Present Value} &= \int_0^{10} 1000te^{-0.03t}\,dt = 1000\cdot\frac{e^{-0.03t}}{(0.03)^2}(-0.03t-1)\Big|_0^{10}\\ &= \frac{1000}{0.0009}\left(e^{-0.3}(-1.3) - e^0(-1)\right)\\ &= \frac{1000}{0.0009}\left(1 - 1.3e^{-0.3}\right) \approx 41040.35\end{aligned}$$

Using the formula $\displaystyle\int ue^{au}\,du = \frac{e^{au}}{a^2}(au-1) + C$, with $a = -0.03$.

3. (6 pts) The population of a small island is growing at a rate that is proportional to the square root of its size. In the year 2000 the island's population was 900, and in the year 2010 the population was 1024. What will the island's population be in 2020?

(i) "*... growing at a rate that is proportional to the square root of its size*" $\Longrightarrow$ $\displaystyle\frac{dP}{dt} = k\sqrt{P}$,
where $P(t)$ is the size of the population at time t (measured in years), and k is the (unknown) constant of proportionality.

(ii) Separate: $\displaystyle\frac{dP}{\sqrt{P}} = k\,dt$

(iii) Integrate: $\displaystyle\int\frac{dP}{\sqrt{P}} = \int k\,dt \Longrightarrow 2\sqrt{P} = kt + C \Longrightarrow \sqrt{P} = kt + C.$

(*) The unknown constants k and C *absorb* the factor 1/2.

(iv) Solve for P: $\sqrt{P} = kt + C \Longrightarrow P = (kt+C)^2$.

(v) Use the data to find k and C, and the population in 2020:

Year 2000: $P(0) = 900 = (k\cdot 0 + C)^2 = C^2 \Longrightarrow C = 30.$[†]

Year 2010: $P(10) = 1024 = (10k+30)^2 \Longrightarrow (10k+30)^2 = 32^2 \Longrightarrow 10k = 2 \Longrightarrow k = 0.2.$

Year 2020: $P(20) = (0.2\cdot 20 + 30)^2 = 34^2 = 1156.$

4. (4 pts) Find the indicated partial derivatives of the function $f(x,y) = \dfrac{x^2y+3xy^3}{4x+1}$.
Clean up your answers.

$$\frac{\partial f}{\partial x} = \frac{(2xy+3y^3)(4x+1) - 4(x^2y+3xy^3)}{(4x+1)^2} = \frac{4x^2y+2xy+3y^3}{(4x+1)^2}$$ (Need to use quotient rule for this one.)

$$\frac{\partial f}{\partial y} = \frac{1}{4x+1}\cdot\frac{\partial}{\partial y}\left(x^2y+3xy^3\right) = \frac{x^2+9xy^2}{4x+1}$$ (No need to use quotient rule for this one.)

[†] It is also possible to choose $C = -30$, which will lead to a different k and a different solution. This particular initial value problem does not have a unique solution — my bad.