1. (6 pts) $\int_1^3 6x \cdot \sqrt[3]{x^2-1}\,dx = \ldots$

Substitute $u = x^2 - 1$, $du = 2x\,dx \implies 6x\,dx = 3\,du$. *Also* $x = 1 \implies u = 0$ *and* $x = 3 \implies u = 8$, *so*

$$\int_1^3 6x \cdot \sqrt[3]{x^2-1}\,dx = 3\int_0^8 u^{1/3}\,du = 3\left(\left.\frac{u^{4/3}}{4/3}\right|_0^8\right) = 3\left(\frac{3}{4}\cdot 8^{4/3} - 0\right) = 36.$$

2. (6 pts) Find the average value of the function $f(x) = \dfrac{4x+1}{2x^{1/3}}$ on the interval $[1, 8]$.

$$\begin{aligned}
\text{Avg}(f) &= \frac{1}{8-1}\int_1^8 \frac{4x+1}{2x^{1/3}}\,dx \\
&= \frac{1}{7}\int_1^8 2x^{2/3} + \frac{1}{2}x^{-1/3}\,dx \\
&= \frac{1}{7}\left(2\cdot\frac{x^{5/3}}{5/3} + \frac{1}{2}\cdot\left.\frac{x^{2/3}}{2/3}\right|_1^8\right) \\
&= \frac{1}{7}\left(\left(\frac{6}{5}\cdot 8^{5/3} + \frac{3}{4}\cdot 8^{2/3}\right) - \left(\frac{6}{5}\cdot 1^{5/3} + \frac{3}{4}\cdot 1^{2/3}\right)\right) \\
&= \frac{1}{7}\left(\frac{192}{5} + 3 - \frac{6}{5} - \frac{3}{4}\right) = \frac{789}{140} \quad (\approx 5.636)
\end{aligned}$$

3. (8 pts) Find the Consumers' and Producers' surplus at equilibrium for the market with supply and demand equations

$$p = 0.05q^2 + q + 10 \quad \text{(supply)} \qquad \text{and} \qquad p = 80 - 1.5q \quad \text{(demand)}.$$

Equilibrium: $0.05q^2 + q + 10 = 80 - 1.5q \implies 0.05q^2 + 2.5q - 70 = 0 \implies \ldots$

$$\ldots \implies q = \frac{-2.5 \pm \sqrt{2.5^2 + 0.2\cdot 70}}{0.1} = \frac{-2.5 \pm 4.5}{0.1} \implies q = 20 \text{ or } q = -70.$$

Since quantity (and price) must both be positive, the equilibrium quantity is $q^* = 20$ *and the equilibrium price is* $p^* = 80 - 1.5\cdot 20 = 50$.

Surpluses:

$$\begin{aligned}
CS = \int_0^{q^*} (\text{Demand} - p^*)\,dq &= \int_0^{20} 80 - 1.5q - 50\,dq \\
&= \left.\left(30q - \frac{3}{4}q^2\right)\right|_0^{20} \\
&= 600 - 300 = 300.
\end{aligned}$$

and

$$\begin{aligned}
PS = \int_0^{q^*} p^* - \text{Supply}\,dq &= \int_0^{20} 50 - \left(0.05q^2 + q + 10\right)\,dq \\
&= \left.\left(40q - \frac{q^2}{2} - \frac{0.05q^3}{3}\right)\right|_0^{20} \\
&= 800 - 200 - \frac{400}{3} = \frac{1400}{3} \approx 466.67.
\end{aligned}$$