1. (6 pts) $\int \frac{3x^2+8x+1}{x^2}\,dx = \int 3 + \frac{8}{x} + x^{-2}\,dx = 3x + 8\ln|x| - x^{-1} + C.$

2. (7 pts) $\int \frac{5x}{(2x^2+1)^3}\,dx = \ldots$

...substitute $u = 2x^2+1$, so $du = 4x\,dx$ which means that $5x\,dx = \frac{5}{4}\,du$, so

$$\int \frac{5x}{(2x^2+1)^3}\,dx = \frac{5}{4}\int u^{-3}\,du = \frac{5}{4}\cdot\frac{u^{-2}}{-2} + C = -\frac{5}{8}(2x^2+1)^{-2} + C.$$

3. (7 pts) A monopolistic firm's marginal revenue function is given by

$$\frac{dr}{dq} = 100 - \sqrt{q+25}.$$

Find the demand equation for this firm's product.

Step one – integrate, using the substitution $u = q + 25$ and $du = dq$:

$$\int 100 - \sqrt{q+25}\,dq = \int 100 - u^{1/2}\,du = 100u - \frac{2}{3}u^{3/2} + C = 100(q+25) - \frac{2}{3}(q+25)^{3/2} + C = 100q - \frac{2}{3}(q+25)^{3/2} + C$$

(because the term $2500 = 100\cdot 25$ is absorbed by C). This means that the firm's revenue function has the form

$$r = 100q - \frac{2}{3}(q+25)^{3/2} + C,$$

where C is an (as-of-yet) unknown constant.

Step two – solve for C. For this we use the fact (assumption, actually) that $r(0) = 0$:

$$0 = r(0) = 100\cdot 0 - \frac{2}{3}(0+25)^{3/2} + C = -\frac{250}{3} + C \implies C = \frac{250}{3}.$$

I.e., the revenue function is

$$r = 100q - \frac{2}{3}(q+25)^{3/2} + \frac{250}{3}$$

Step 3 – Find the demand equation, using the relation $r = pq$, so $p = \frac{r}{q}$:

$$p = \frac{r}{q} = \frac{100q - \frac{2}{3}(q+25)^{3/2} + \frac{250}{3}}{q} = 100 - \frac{2(q+25)^{3/2} + 250}{3q}.$$